USING LATIN HYPERCUBE SAMPLING TECHNIQUE FOR MULTIOBJECTIVE OPTIMIZATION OF WATER SUPPLY SYSTEM DESIGN UNDER UNCERTAINTY

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SUMMARY

Water demand uncertainty in a water supply system (WSS) arises mainly due to the abnormal behaviors of water users and the change of network configuration when it is expanded to new consumers. In practice, this directly impacts the optimal designed WSS. This paper presents a methodology to address the issue of water demand uncertainty in the designing a WSS that combines the Latin Hypercube Sampling Technique (LHST) and a multiobjective algorithm optimization. The two objectives are: (1) minimisation of capital cost, and (2) maximization of WSS robustness. The decision variables are the pipe diameter alternatives for each pipe in the network under constraints of nodal head limitations. The output from the multiobjective algorithm optimization process is the Pareto front containing design solutions which are the trade-off solutions in terms of the two objectives. Both cases of uncertain uncorrelated and correlated demand were taken into account. The new methodology is tested on two benchmark published water supply systems: Two loop network and Hanoi network. With only thousand samples, the LHST was capable of producing a good range of random output variables corresponding to uncertain input variables. The result will support more options for designers to select the most appropriate network configuration and it is clear that neglecting demand uncertainty may lead to a seriously under-designed network.

Keywords: LHST, multiobjective optimization, uncertainty, WSS design.

1. INTRODUCTION

The aim of designing a water supply system (WSS) is to provide sufficient water to consumers over a long period of time meeting performance requirements such as required quantity, quality, and pressure at nodes with lower cost and higher system robustness. Unfortunately, a number of uncertainties exist in the operation process as abnormal operating conditions such as water demand, pipe roughness, component failure, and pressure requirement (Chung et al., 2009; Basupi and Kapelan, 2015; Thissen et al., 2017…).

The most notable source amongst uncertainties in WSS design is water demand at nodes and it arises mainly due to the different behaviors of water users and the change of network configuration when it is expanded to new consumers as well. Water demand uncertainty directly impacts the uncertainty in nodal pressure head as well as other hydraulic parameters, therefore within an optimal WSS design procedure, studying uncertain conditions which impacts network reliability has received considerable attention in the research community (Babayan et al., 2005; Kapelan et al., 2005, Sun et al., 2011...).

Babayan et al. (2005) developed a new approach where the standard genetic algorithm (GA) is linked with Epanet (Rossman, 2000) to an integration-based uncertainty quantification method. In the study, the uncertain demand was assumed to follow the normal probability density function (PDF) with a predefined standard deviation of 10% from mean value. The network reliability was then determined using a Monte Carlo simulation (MC) with large number of samples. The results compared to available deterministic solutions demonstrated the importance of applying the uncertainty concept in WSS optimization. However, the level of robustness of the designed network was not estimated directly and explicitly.

Kapelan et al. (2005) assumes that a lot of information is required to define probability density functions of input parameters by using MC and, therefore, a lot of time is consumed. Hence, the Latin hypercube sampling technique (LHST) was used in the multi-objective optimization framework to identify the optimal robust Pareto fronts of minimizing the cost and maximizing the robustness. A small number of samples were enough for each objective evaluation leading to significant computational savings when compared to the full sampling approach.

Sun et al. (2011) proposed a fast approach to improve computational efficient when addressing the multi-objective WSS design optimization including cost and robustness under uncertain nodal demands. Compared to traditional methods (MC and LHST) the fast approach saves a large amount of computational time but it produces somewhat more expensive designs, particularly in the part of the Pareto front where the solutions have a robustness greater than 80%.

In real WSSs, nodal demands are highly correlated due to abnormal conditions, e.g., hot, dry weather that affects the network as a whole. For solving this situation, nodal demands were assumed to be Gaussian distributed and temporally correlated at a coefficient of 0.5 between any two nodes. Sampling technique is the approach to generate series of random uncertain input variables (nodal demands). This procedure can be obtained by many ways depending mainly upon the distribution of a sample set. Since demand changes may follow the normal distribution (Babayan et al., 2005; Kapelan et al., 2005), MC simulation has been used frequently. However, this technique is time consuming since it requires a very large number of samples (hundred thousands) to produce an acceptable result. In contrast, the LHST needs only small number of samples. A dominant advantage of the LHST, compared to MC, is a better random sample stratification, i.e. nodal demands, which leads to a more accurate evaluation of the nodal pressures.

In practice, nodal demands in a WSS can also be not independent because they may temporally depend on the scale of some factors which affect the network as a whole. For instance, hot and dry weather can result in a significant extra consumption for all nodes. Hence, LHST with a procedure proposed by Iman and Conover (1982) will be employed to produce a rank correlation matrix of uncertain correlated demand variables in this study.

In this study, a network robustness probability is defined to evaluate the network robustness under demand uncertainty. The new methodology is tested on two benchmark published water supply networks: Two loop network and Hanoi network.

2. RESEARCH METHODOLOGY

2.1. Multiobjective problem definition

The aim of using single objective WSS optimization is to find either the “least cost” or “most benefit” solution. However, a frequently asked question arising from this is how reliable the behavior of the least-cost designed WSS will be in case any failure or uncertainty occurs. Therefore, in this study, the objectives of the robust design methodology presented here are to (1) minimize the capital cost and (2) maximize the system robustness. The robustness is defined here as the probability that heads at all network nodes are simultaneously equal to or above the corresponding minimum requirements for that node. More specifically, the optimization problem is formulated as follows:

$$Minimize\ PIC = \sum_{i=1}^{np} C(D_i).L_i \quad (1)$$

$$Maximize\ RP = P(H_i \geq H_{i,req}; \forall_i = 1, \dots, N_n) \quad (2)$$

where *PIC* is total capital cost, $C(D_i)$ is unit cost per unit length of pipe with diameter D_i, L_i is pipe length, D_i is commercial pipe diameter i^{th}; $D_i \in D$, with D is discrete set of all available design options, *np* is number of pipe in the system, and *RP* is design robustness defined as probability *P* that heads (H_i) at all system nodes are simultaneously equal to or above the corresponding minimum requirements for that node ($H_{i,req}$). The set of commercial pipe diameters are considered as decision variables in a process of optimization, then the optimal design procedure will select alternative pipe diameters taken only out of this set with constraints of nodal head limitations.

2.2. An approach for solving uncertain demand problem

Water demand uncertainty arises mainly due to the different behaviors of water users and the change of network configuration when it is expanded to new consumers. In practice, nodal demands in a WSS may temporally depend on the scale of some factors which affect the network as a whole. For instance, hot and dry weather can result in a significant extra consumption for all nodes. Therefore, LHST with a procedure (Iman and Conover, 1982) will be employed to create a rank correlation matrix of uncertain correlated demand variables in this study. The procedure to produce correlation matrix of nodal demands can be briefly described as follows:

Suppose that *R (Ns x Nv)* as the matrix of independent random samples is generated based on mean and standard deviation. *Ns* is the number of samples and *Nv* is the number of uncertain nodal demand input variables.

Let *C (Nv x Nv)* be the desired correlation matrix with the desired correlation coefficient. Because correlation matrix *C* is positive definite and symmetric, it may be written as C = PP' where *P* is the lower triangular matrix obtained by using Cholesky factorization.

Matrix *R** with the desired correlation coefficient is achieved as *R* = RP'*. The rank correlation matrix *M* of *R** should be close to *C*.

The samples in *R* are finally rearranged column-wise to have the same rank ordering as the corresponding column of *R**. Thus, the input values have the same sample rank correlation matrix that *R** has. Each row of input values matrix now represents a single correlated demand loading condition.

By using this procedure, a set of random uncertain input nodal demands are generated based on a set of expected values (or mean values - μ) and standard deviations (σ). Expected values are assumed equaling to the deterministic demands at nodes. While standard deviations are hypothesized equaling to 10% and 30% of the corresponding expected values.

The uncertain output variables (nodal heads or pressures) corresponding to these uncertain input variables are then calculated using the Epanet model. Subsequently, the robustness probability is computed as the ratio (percentage) of the number of times (N_C) that a particular criterion is satisfied at all nodes (i.e. nodal pressures are not smaller than corresponding minimum required pressures) to total number of samples (N_S) as given in Eq.3:

$$RP = \frac{N_C}{N_S} \tag{3}$$

Where *RP* is so-called the robustness probability of network; N_C is number of times that the minimum required heads are met simultaneously at all nodes.

Procedure solving the WSS optimal design under demand uncertainty is expressed in Fig. 1 and can be interpreted as follows:

(i) From the Pareto front achieved by multiobjective optimization, several significant solutions are proposed.

(ii) Nodal demands are the only uncertain variables, which are assumed following a normal distribution with means equal to the deterministic demands at demand nodes and an assumed standard deviation.

(iii) The LHST is in turn executed with the proposed solution to generate a matrix including *Ns* rows and *Nv* columns of random uncertain input nodal demands. Based on either uncertain uncorrelated demands or uncertain correlated demands, a different LHST procedure is used.

(iv) Corresponding to each row of input nodal demand matrix, a set of corresponding nodal heads is evaluated using Epanet model. Finally, a matrix of random nodal heads is evaluated including *Ns* rows and *Nv* columns.

(v) Calculate network robustness probability equivalent to the network configuration using Eq.3. Associate the robustness probability with cost and network reliability the results will support designers to select an appropriate solution under demand uncertainty.

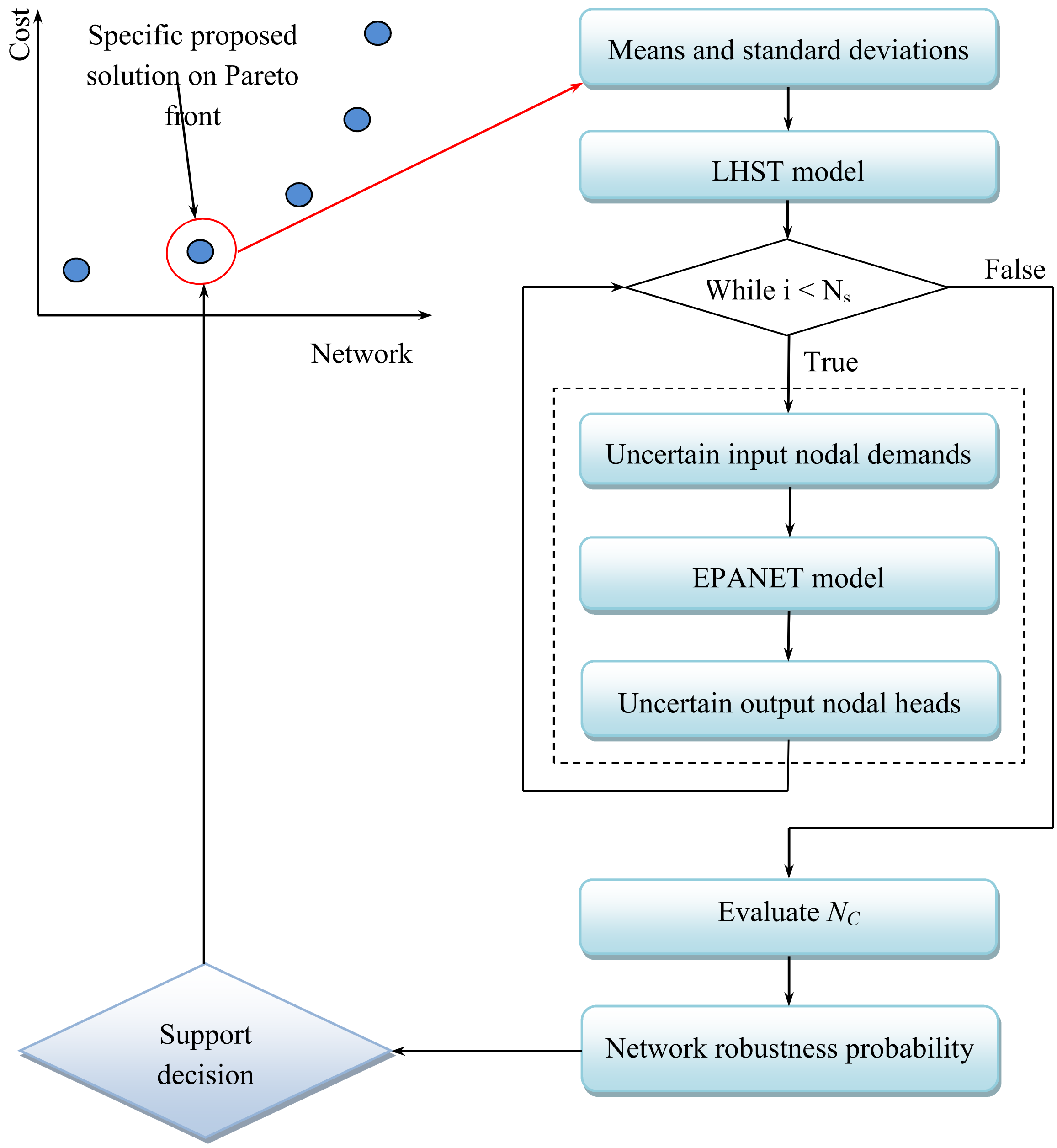

Figure 1. Procedure for evaluating robustness probability of WSS optimal design under demand uncertainty

3. RESULTS AND DISCUSSION

Demand uncertainties can be either uncorrelated or correlated as discussed above. Uncorrelated demand occurs whenever an uncertain demand taking place at a node does not impact the others; otherwise it is the case of correlated demand. It is assumed here that nodal demands are:

(1) Uncertain uncorrelated demands with a standard deviation equal to 10% of the mean value (i.e., coefficient of variation $\sigma = 0.1\mu$);

(2) Uncertain uncorrelated demands with a standard deviation equal to 30% of the mean value (i.e., coefficient of variation $\sigma = 0.3\mu$);

(3) Uncertain correlated demands with a standard deviation equal to 10% of the mean value (i.e., coefficient of variation $\sigma = 0.1\mu$).

The capability of the proposed approach in this study was evaluated for both uncorrelated or correlated uncertainty based on its application on two benchmark published networks: Two loop network (TLN) and Hanoi network (HN), the network description can be found in Geem, Z. W. (2006).

3.1. Uncertain uncorrelated demands

3.1.1. Application on TLN water supply system

Figure 2 express all trade-offs Pareto solutions produced by the multiobjetive optimization procedure between the total capital cost and the network robustness probability. The red circles represent possible solutions with a standard deviation equal to 10% of the mean value or coefficient of variation $\sigma = 0.1\mu$ (Case 1), while the blue x-marks represent possible solutions with coefficient of variation $\sigma = 0.3\mu$ (Case 2).

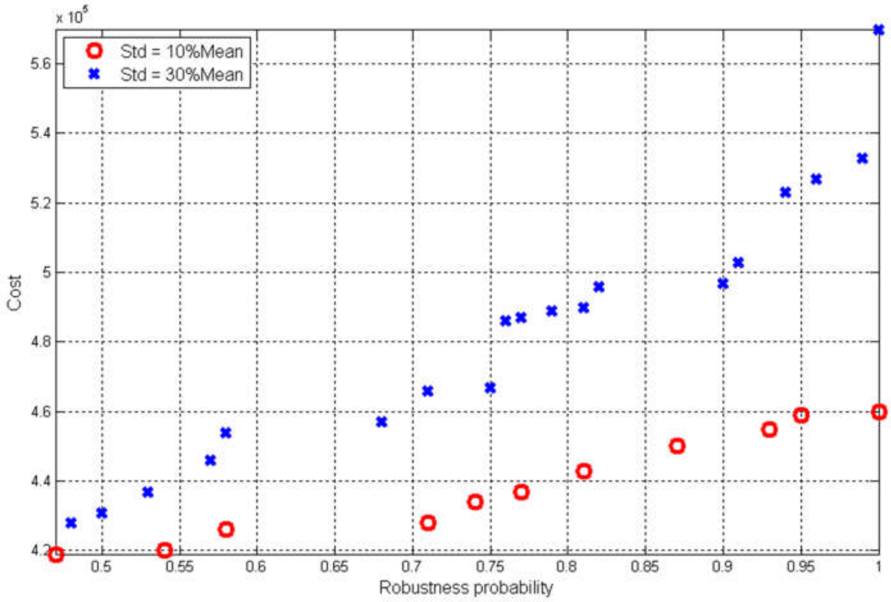

Figure 2. Pareto solutions obtained by multiobjetive optimization with respect to the TLN

Applying the new approach for solving uncertain demand problem to several solutions selected on the Pareto optimal front based on cost, the robustness probabilities for TLN were achieved (Table 1). To obtain theses, three different sample sizes of (i) 1,000; (ii) 10,000; and (iii) 100,000 samples were used in order to verify the stunning performance of LHST. The robustness probability value decreases when the demand uncertainty increases, or in other words, robustness probability rose with the higher cost and network reliability as well.

Table 1. Robustness probability under uncertain uncorrelated demands corresponding to different cost solutions and different samples (i:1,000 samples; ii: 10,000 samples, and iii: 100,000 samples)

Solutions	Cost ($)	Robustness probability with demand uncertainty (%)					
		Case 1: $\sigma = 0.1\mu$			Case 2: $\sigma = 0.3\mu$		
		i	ii	iii	i	ii	iii
Solution 1	419,000	45.1	45.15	45.08	N/A	N/A	N/A
Solution 2	428,000	58.7	58.68	58.51	46.8	46.82	46.85
Solution 3	438,000	74.6	74.56	74.62	N/A	N/A	N/A
Solution 4	452,000	87.25	87.3	87.22	N/A	N/A	N/A
Solution 5	455,000	N/A	N/A	N/A	57.82	57.79	57.85
Solution 6	460,000	100	100	100	N/A	N/A	N/A
Solution 7	486,000	100	100	100	76.45	76.4	76.48
Solution 8	522,000	100	100	100	94.22	94.2	94.18
Solution 9	570,000	100	100	100	100	100	100
Average running time: (i) 2.10; (ii) 23.4; and (iii) 910.0 seconds, respectively (computer: AMD Dual Core 2.0 GHz)							

Note: N/A is not available.

In the table 1, there are several solutions representing cost and corresponding robustness probability which selected randomly from the Pareto fronts produced from WSS optimization in case of uncertain uncorrelated demands with $\sigma = 0.1\mu$ or $\sigma = 0.3\mu$. The Table also displays that there were only little difference of RP values but big difference of calculation time among the results produced by three big numbers of samples. Consequently, the number of samples of 1,000 can be large enough to reflect the precise results. Also, it can reduce a considerable computational time. Therefore, this value would be used for calculating robustness probability of HN benchmark system.

3.1.2. Application on HN water supply system

All trade-offs Pareto solutions produced by the multiobjetive optimization procedure between the total capital cost and the network robustness probability for HN system is displayed on figure 3 with respect to 2 cases (Case 1 and Case 2).

By applying the procedure mentioned in section 2.2 on several proposed solutions for Hanoi network, the robustness probability with 1,000 samples can be achieved as presented in table 2.

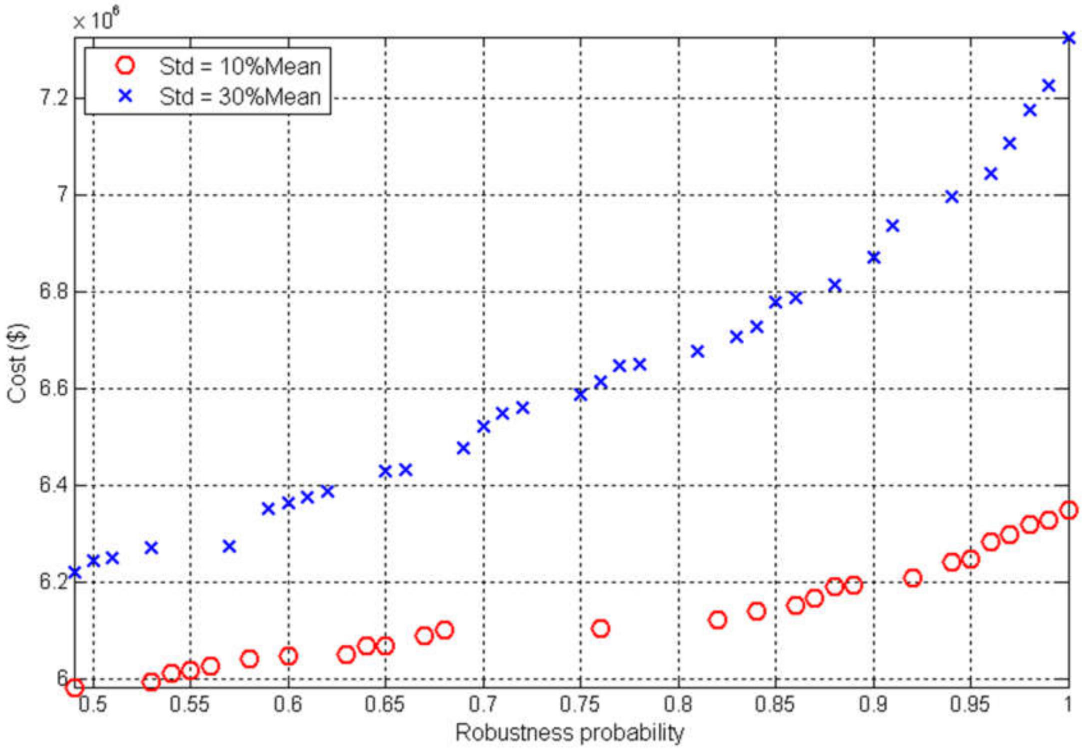

Figure 3. Pareto solutions obtained by multiobjetive optimization with respect to the HN

Table 1. Robustness probability under uncertain uncorrelated demands corresponding to different cost solutions with 1,000 samples

Solutions	**Cost (Mi.$)**	**Robustness probability with demand uncertainty (%)**	
		$\sigma = 0.1\mu$	$\sigma = 0.3\mu$
Solution 1	6.046	48.2	N/A
Solution 2	6.152	68.1	N/A
Solution 3	6.194	88.0	N/A
Solution 4	6.218	N/A	48.2
Solution 5	6.370	100	58.6
Solution 6	6.595	100	75.0
Solution 7	6.780	100	85.0
Solution 8	7.320	100	100

From table 1 and table 2, it can be shown that, with small capital cost, robustness probability of water supply system is also small; and the equivalent robustness probability was higher in case of smaller standard deviation (Case 1 with $\sigma = 0.1\mu$). In other words, for the same robustness probability, Case 1 with lower demand fluctuation was more cost-effective than Case 2 ($\sigma = 0.3\mu$) with higher demand fluctuation.

3.2. Uncertain correlated demands (Case 3)

This situation occurs due to the change of the amount of water requirement at the same time, on the whole system, for example, the effect of hot, dry weather to all domestic demands. All nodal demands were assumed to follow normal distribution and their standard deviations were also assumed to equal 10%. The correlation coefficient between demand nodes was assumed to equal 0.5 to investigate its impact. In this circumstance, the correlation matrix *C* (mentioned in section 2.2) was created with the desired correlation coefficient, i.e. the correlation matrix of 1.0 on the main diagonal and 0.5 elsewhere.

The corresponding robustness probabilities to five proposed solutions for TLN and HN network were also estimated using the Latin hypercube sampling technique. Comparing the results derived from 2 cases having the same standard deviation, i.e. Case 1 and Case 3 (Figure 4), showed that:

For the lower cost interval, in general, the robustness probabilities achieved in Case 3 are higher than the corresponding solutions obtained in Case 1. In contrast, for the higher cost interval, the robustness probabilities in Case 3 tend to be lower than in Case 1, or in other words, they are more costly than the corresponding solutions in Case 1. From this comparison, it can be said that a consequence of the correlation between nodal demands was acknowledged. Accordingly, the impact of uncertain demand is more serious in case of lower cost and vice versa.

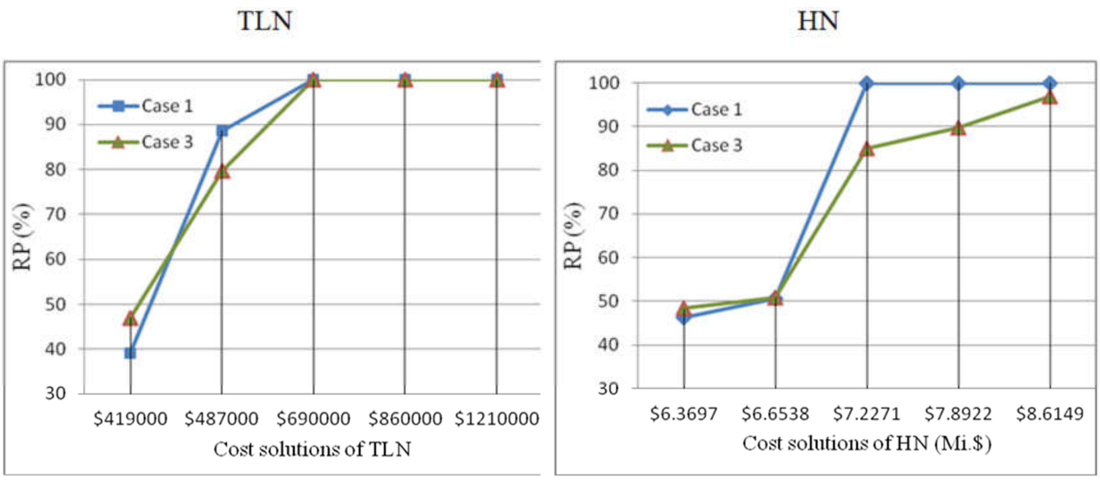

Figure 4. The comparison of robustness probabilities under uncertain uncorrelated and correlated demands (Case 1 and Case 3) for five representative solutions of TLN and HN systems

As a result of two applications above, the multi-objective optimization with respect to WSS design problem is assured to provide more alternatives for a network configuration. The selected network configuration based on either cost may not be advanced because the designers tend to select the network with less cost. To guarantee the configuration under demand uncertainty, it is necessary to consider the network robustness. Therefore, a demand uncertainty consideration may become useful for a more reliable WSS design.

4. CONCLUSION

This paper focused on multiple objective optimizations for optimally designing and operating a WSS towards the consideration of demand uncertainty in WSSs. The two objectives are: (1) minimisation of total cost, and (2) maximization of WSS robustness. The decision variables are the pipe diameter alternatives. The output from the multiobjective algorithm optimization process is the Pareto front containing design solutions which are trade-off solutions in terms of the two objectives.

Uncertain conditions were taken into consideration in order to predict the behavior of the designed network. Among the uncertainties, uncertain nodal demand is the most important one because it directly affects other hydraulic parameters. Normally, demands can be considered as uncertain uncorrelated. However, in some extreme cases, such as hot and dry weather, demands can be increased at all nodes and are then considered uncertain correlated. Both cases were taken into account. With only thousand samples, the Latin hypercube sampling technique was capable of producing a good range of random output variables corresponding to uncertain input variables. The result will support more options for designers to select the most appropriate network configuration and it is clear that neglecting demand uncertainty may lead to a seriously under-designed network.

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SỬ DỤNG THUẬT TOÁN LATIN HYPERCUBE SAMPLING TỐI ƯU ĐA MỤC TIÊU HỆ THỐNG CẤP NƯỚC TRONG ĐIỀU KIỆN BẤT THƯỜNG

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TÓM TẮT

Sự bất thường về nhu cầu dùng nước trong hệ thống cấp nước thường xảy ra khi có hành vi sử dụng nước khác thường hoặc có sự mở rộng hệ thống cấp nước tới người dùng mới. Điều này ảnh hưởng trực tiếp tới chế độ thủy lực của hệ thống đã được thiết kế tối ưu. Bài báo trình bày phương pháp kết hợp giữa thuật toán Latin Hypercube Sampling (LHST) với thuật toán tối ưu đa mục tiêu nhằm giải quyết sự bất thường trong nhu cầu sử dụng nước khi thiết kế hệ thống cấp nước. Các hàm mục tiêu bao gồm: (1) tối thiểu chi phí đầu tư và (2) tối đa độ ổn định của hệ thống cấp nước. Các biến quyết định là các sự lựa chọn đường kính cho các đoạn ống dưới các ràng buộc về cột áp tại các điểm lấy nước. Kết quả quá trình tối ưu đa mục tiêu là biên Pareto bao gồm các giải pháp hài hòa giữa chi phí và độ ổn định của hệ thống. Hai trường hợp bất thường không phụ thuộc và có phụ thuộc lẫn nhau giữa các điểm dùng nước được xem xét trong nghiên cứu này. Mô hình đề xuất được kiểm chứng bởi hai hệ thống cấp nước mẫu: Two loop network and Hanoi network. Chỉ với cỡ mẫu nhỏ (1000) thuật toán LHST cho kết quả tốt với các biến đầu vào bất thường. Kết quả này sẽ hỗ trợ các nhà thiết kế có được sự lựa chọn phù hợp hơn trong việc cân nhắc vốn đầu tư và độ ổn định của hệ thống cấp nước.

Từ khóa: Điều kiện bất thường, LHST, thiết kế hệ thống cấp nước, tối ưu đa mục tiêu.

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